MATHIAS LIKE CRITERION FOR THE EXTENDER BASED PRIKRY FORCING

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Abstract. We suggest a genericity criterion for the extender based Prikry forcing analogous to the Mathias genericity criterion for Prikry forcing.

1. Introduction

Let P_U be Prikry forcing [5] using the measure U. Let $G \subseteq P_U$ be generic. From the generic filter we build the Prikry sequence $\langle \tau_n \mid n < \omega \rangle$ which is the increasing enumeration of the set $\bigcup \{ t \mid \langle t, A \rangle \in G \}$. We can work backwards and generate from the Prikry sequence the generic filter which is the set $\{ \langle \langle \tau_0, \dots, \tau_{k-1} \rangle, A \rangle \mid k < \omega,\ A \in U,\ A \supseteq \{ \tau_n \mid k \leq n < \omega \} \}$. The construction of a filter from an arbitrary sequence as above is possible, but the filter generated will not be necessarily generic. In Mathias [2] a criterion on a sequence of ordinals which is equivalent to the genericity of the filter was presented. For this let us define the following notion. The sequence $\langle \tau_n \mid n < \omega \rangle$ *generates* the measure U if for each set $A \subseteq \kappa$, $A \in U \iff \exists k < \omega\ A \supseteq \{ \tau_n \mid k \leq n < \omega \}$. Now we can quote the Mathias criterion for genericity.

Theorem (Mathias [2])**.** *The following are equivalent:*

(1) The sequence $\langle \tau_n \mid n < \omega \rangle$ is P_U-generic.
(2) The sequence $\langle \tau_n \mid n < \omega \rangle$ generates the measure U.

The aim of this note is to suggest a genericity criterion for the extender based Prikry forcing [1].

Let E be a $\langle \kappa, \lambda \rangle$-extender. Let $\mathbb{P}$ be the extender based Prikry forcing using the extender E. Since $\mathbb{P}$ adds ω-sequences for each $\alpha \in \lambda \setminus \kappa$, we will use a function $F : \lambda \setminus \kappa \to [\kappa]^\omega$ to describe these sequences. Thus for each ordinal $\alpha \in \lambda \setminus \kappa$ let $F(\alpha)$ be the increasing enumeration of $\bigcup \{ f(\alpha) \mid \alpha \in \operatorname{dom} f,\ \langle f, A \rangle \in G \}$, where $G \subseteq \mathbb{P}$ is generic.

From this point on we use notation from the extender based Prikry papers which we give in section 2. Working backwards assume that we are given a function $F : \lambda \setminus \kappa \to [\kappa]^\omega$. Construction of a filter from the function F needs to be done with care since the sequences are not independent of each other (e.g., they code a scale). Using the construction of the generic from the Prikry sequence as a guideline, we put a condition $\langle f, A \rangle \in \mathbb{P}$ in the filter if there is an increasing sequence $\langle \tau_n \mid n < \omega \rangle$ such that $F(\alpha) = \bigcup \{ f_{\langle \tau_0, \dots, \tau_n \rangle}(\alpha) \mid n < \omega \}$, $A \supseteq \{ \tau_n \mid n < \omega \}$, and $\langle \tau_n \mid n < \omega \rangle$ generates the measure $E(\operatorname{dom} f)$. If the generated filter is $\mathbb{P}$-generic then we will say

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the function F is $\mathbb{P}$-generic. However, the assumptions in the above construction of the filter are not enough to guarantee genericity, which is to be expected as there is a (not so) hidden Cohen forcing in the extender based Prikry forcing. Thus we proceed as follows.

For a large enough regular cardinal χ we say the elementary substructure $N \prec H(\chi)$ is *appropriate* if $|N| = \kappa$, $N \supseteq {}^{<\kappa}N$, and $\mathbb{P} \in N$. We say that the function F is N*-generic* if there is a condition $f \in \mathbb{P}^*$ and an increasing sequence $\langle \tau_n \mid n < \omega \rangle$ such that $F(\alpha) = \bigcup \{ f_{\langle \tau_0, \dots, \tau_{n-1} \rangle}(\alpha) \mid n < \omega \}$ for each $\alpha \in (\lambda \setminus \kappa) \cap N$, the sequnce $\langle \tau_n \mid n < \omega \rangle$ generates the measure $E(\operatorname{dom} f)$, f is $\langle N, \mathbb{P}^* \rangle$-generic, and $\operatorname{dom} f \subseteq N$. ($\mathbb{P}^*$ is the projection of $\mathbb{P}$ to the first coordinate.)

Now we can state the genericity criterion, proved in *lemma* 3.3 and *lemma* 3.4 of this paper.

Theorem. *The following are equivalent:*
(1) *The function* $F : \lambda \setminus \kappa \to [\kappa]^\omega$ *is* $\mathbb{P}$*-generic.*
(2) *The set* $\{ N \in V \mid (N \prec H(\chi))_V,\ F \text{ is } N\text{-generic} \}$ *is unbounded in* $(H(\chi)^{\leq\kappa})_V$.

The structure of this note is as follows. In section 2 we present the extender-based Prikry forcing using the notation of [4]. In section 3 we prove the theorem.

We assume knowledge of the extender based Prikry forcing throughout this note.

2. Preliminaries

In this section we present the extender based Prikry forcing and quote two facts about it that we need. The form of the definition we give is a special case of the definitions from [4]. As for the facts, we refer to the proofs in [3] (where the notation is somewhat archaic) and not to [4] (where the situation is too complicated for our needs).

Throughout this note let E be a $\langle \kappa, \lambda \rangle$-extender and $j : V \to M \simeq \operatorname{Ult}(V, E)$ be the natural embedding of V into the ultrapower M.

In the forcing notion we need sets which are measure one in the sense of several measures at once. Let $d \in [\lambda \setminus \kappa]^{<\kappa}$ be an increasing sequence of ordinals. We could have defined a measure $E(d)$ by letting $A \in E(d) \iff d \in j(A)$. Then we could have assumed that if $\nu \in A \in E(d)$ then $\nu \subseteq \kappa$ is an increasing sequence of ordinals such that $\operatorname{ot}(\nu) = \operatorname{ot}(d)$. We could also compute from ν and d to which index in the extender corresponds an ordinal $\xi \in \nu$: It will correspond to $\alpha \in d$ satisfying $\operatorname{ot}(d \cap \alpha) = \operatorname{ot}(\nu \cap \xi)$.

However, we need to use κ-many measures at once. If we take $d \in [\lambda \setminus \kappa]^\kappa$ we still could have defined $E(d)$ as above. In this case, however, finding to which measure an ordinal $\xi \in \nu$ corresponds becomes rather cumbersome. We solve this by defining $E(d)$ as follows for each $d \in [\lambda \setminus \kappa]^{\leq\kappa}$:

$$A \in E(d) \iff \langle \langle j(\alpha), \alpha \rangle \mid \alpha \in d \rangle \in j(A).$$

Thus if $\nu \in A \in E(d)$ then ν is typically a function and the ordinal $\nu(\alpha)$ corresponds to the extender index α. We will use sets d such that $\kappa \in d$. A set $A \in E(d)$ might contain a measure zero set of functions ν with an erratic behavior. Thus we will use sets from $E(d)$ which are good in the following sense.

Definition 2.1. A set $A \in E(d)$ is good if for each $\nu, \mu \in A$ the following hold:
(1) ν is a strictly increasing function.
(2) $\kappa \in \operatorname{dom} \nu \subseteq d$.

(3) $\operatorname{ran}\nu \subseteq \kappa$.
(4) $|\nu| \leq \nu(\kappa)$.
(5) If $\nu(\kappa) = \mu(\kappa)$ then $\operatorname{dom}\nu = \operatorname{dom}\mu$.

Note that the good subsets are dense in $E(d)$ in the following sense. If $A \in E(d)$ then there is a good set $B \in E(d)$ such that $B \subseteq A$.

If $A \in E(d)$ is a good set and $\nu, \mu \in A$ then we say that ν is below μ (denoted $\nu < \mu$) if $\operatorname{dom}\nu \subseteq \operatorname{dom}\mu$ and $\nu(\alpha) < \mu(\kappa)$ for each $\alpha \in \operatorname{dom}\nu$.

The definition of the forcing notion $\mathbb{P}$ begins in the following definition and ends in definition 2.5.

Definition 2.2 (Conditions)**.** A condition in the forcing notion $\mathbb{P}$ is of the form $\langle f, A\rangle$, where the following hold:
(1) $f : d \to [\kappa]^{<\omega}$ is a function such that $d \in [\lambda \setminus \kappa]^{\leq\kappa}$ and $\kappa \in d$.
(2) $A \in E(d)$ is a good set.
(3) For each $\nu \in A$ and $\alpha \in \operatorname{dom}\nu$, $\max f(\alpha) < \nu(\kappa)$.

As is customary, if $p = \langle f, A\rangle$ is a condition then we denote f and A by f^p and A^p, respectively.

Definition 2.3 (Direct order)**.** The condition q is a direct extension of the condition p, denoted either $q \leq^* p$ or $q \leq^0 p$, if $f^q \supseteq f^p$ and $A^q \restriction \operatorname{dom} f^p \subseteq A^p$, where $A^q \restriction \operatorname{dom} f^p = \{\nu \restriction \operatorname{dom} f^p \mid \nu \in A^q\}$.

Definition 2.4 (Extension by ν)**.** Let $f : d \to [\kappa]^{<\omega}$ be a function. Let ν be a function such that $\operatorname{dom}\nu \subseteq d$ and $\operatorname{ran}\nu \subseteq \kappa$. The function $f_{\langle\nu\rangle} : d \to [\kappa]^{<\omega}$ is defined as follows for each $\alpha \in d$,

$$f_{\langle\nu\rangle}(\alpha) = \begin{cases} f(\alpha)^\frown\langle\nu(\alpha)\rangle & \alpha \in \operatorname{dom}\nu \text{ and } \forall\beta \in \operatorname{dom}\nu \ \max f(\beta) < \nu(\kappa), \\ f(\alpha) & \text{otherwise.} \end{cases}$$

Assume $\langle f, A\rangle$ is a condition and $\nu \in A$. Then the 1-point extension of $\langle f, A\rangle$ by ν is the condition $\langle f, A\rangle_{\langle\nu\rangle} = \langle f_{\langle\nu\rangle}, A_{\langle\nu\rangle}\rangle$, where $A_{\langle\nu\rangle} = \{\mu \in A \mid \nu < \mu\}$.

By recursion define $\langle f, A\rangle_{\langle\nu_0,\dots,\nu_n,\nu_{n+1}\rangle} = (\langle f, A\rangle_{\langle\nu_0,\dots,\nu_n\rangle})_{\langle\nu_{n+1}\rangle}$.

Definition 2.5 (Order)**.** Assume $n < \omega$. The condition q is an $n{+}1$-point extension of the condition p, denoted $q \leq^{n+1} p$, if there is $\nu \in A^p$ such that $q \leq^n p_{\langle\nu\rangle}$.

The condition q is an extension of the condition p, denoted $q \leq p$, if there is $n < \omega$ such that $q \leq^n p$.

Claim 2.6 (The strong Prikry property, [3] theorem 3.25)**.** *Assume $p \in \mathbb{P}$ is a condition and D is a dense open subset of $\mathbb{P}$. Then there is a direct extension $p^* \leq^* p$ and $n < \omega$ such that for each $\langle\nu_0,\dots,\nu_{n-1}\rangle \in [A^{p^*}]^n$, $p^*_{\langle\nu_0,\dots,\nu_{n-1}\rangle} \in D$.*

We denote by $\mathbb{P}^*$ the projection of $\mathbb{P}$ to the first coordinate, i.e., $\mathbb{P}^* = \{f \mid \langle f, A\rangle \in \mathbb{P}\}$. The order on $\mathbb{P}^*$ is reverse inclusion, i.e., $f \leq g \iff f \supseteq g$. Note that we do not force with $\mathbb{P}^*$.

Let P be some forcing notion. Let $N \prec H(\chi)$ be an elementary substructure such that $P \in N$. We say that a condition $p \in P$ is $\langle N, P\rangle$-generic if for each dense open subset $D \subseteq P$ which is in N we have $p \Vdash$ "$D \cap \underset{\sim}{G} \cap N \neq \emptyset$", where $\underset{\sim}{G}$ is the name of the P-generic filter.

Claim 2.7. *Assume that $N \prec H(\chi)$ is an appropriate elementary substructure, f is $\langle N, \mathbb{P}^*\rangle$-generic and* $\operatorname{dom} f \subseteq N$. *Then there is an $\langle N, \mathbb{P}\rangle$-generic condition $p \in \mathbb{P}$ such that $f^p = f$.*

Proof. Evident from the proof of [3, Claim 3.29]. □

3. The Genericity Criterion

The motivation for the following definition is Lemma 3.3 below.

Definition 3.1. Assume that $N \prec H(\chi)$ is an appropriate elementary substructure. We say the function $F : \lambda \setminus \kappa \to [\kappa]^\omega$ is N-generic if there is a function $f : d \to [\kappa]^{<\omega} \in \mathbb{P}^*$, where $d = (\lambda \setminus \kappa) \cap N$, and an increasing sequence $\langle \tau_n \mid n < \omega \rangle$ such that the following hold:

(1) f is $\langle N, \mathbb{P}^* \rangle$-generic and $\operatorname{dom} f \subseteq N$.
(2) The sequence $\langle \tau_n \mid n < \omega \rangle$ generates the measure $E(d)$.
(3) For each $\alpha \in d$, $F(\alpha) = \bigcup \{ f_{\langle \tau_0, \dots, \tau_n \rangle}(\alpha) \mid n < \omega \}$.

Definition 3.2. Assume that G is $\mathbb{P}$-generic. Define the function $F_G : \lambda \setminus \kappa \to [\kappa]^\omega$ by setting for each $\alpha \in \lambda \setminus \kappa$, $F_G(\alpha) = \bigcup \{ f(\alpha) \mid \langle f, A \rangle \in G \}$. Denote by $\dot{F}_G$ the $\mathbb{P}$-name of F_G.

Lemma 3.3. *(In $V[G]$) The set $\{ N \in V \mid (N \prec H(\chi))_V,\ F_G \text{ is } N\text{-generic} \}$ is unbounded in $(H(\chi)^{\leq \kappa})_V$.*

Proof. Work in V. Let $X \in [H(\chi)]^{\leq \kappa}$ be arbitrary. Fix a condition $p \in \mathbb{P}$. Choose an appropriate elementary substructure $N \prec H(\chi)$ such that $N \supseteq X \cup \{p\}$. Let $f^* \in \mathbb{P}^*$ be an $\langle N, \mathbb{P}^* \rangle$-generic condition such that $f^* \supseteq f^p$ and $\operatorname{dom} f^* \subseteq N$. By claim 2.7 there is an extension $p^* \leq^* p$ which is $\langle N, \mathbb{P} \rangle$-generic such that $f^{p^*} = f^*$.

Thus, by a density argument we can find a condition $p^* \in G$ and an appropriate elementary substructure $N \prec H(\chi)$ so that $X \subseteq N$, f^{p^*} is $\langle N, \mathbb{P}^* \rangle$-generic, $\operatorname{dom} f^{p^*} \subseteq N$, and p^* is $\langle N, \mathbb{P} \rangle$-generic.

Then in $V[G]$ there is an increasing sequence $\langle \tau_n \mid n < \omega \rangle \subseteq A^{p^*}$ such that $p^*_{\langle \tau_0, \dots, \tau_n \rangle} \in G$ for each $n < \omega$. By definition, $F_G(\alpha) = \bigcup \{ f^{p^*}_{\langle \tau_0, \dots, \tau_n \rangle}(\alpha) \mid n < \omega \}$ for each $\alpha \in d$, where $\alpha \in (\lambda \setminus \kappa) \cap N$. We are left with showing that $\langle \tau_n \mid n < \omega \rangle$ generates $E(d)$.

Assume $A \in E(d)$. By a density argument there is an extension $q \leq p^*$ such that $q \in G$ and $A^q \restriction \operatorname{dom} f^{p^*} \subseteq A$. Hence there is $k < \omega$ such that $q \leq^* p^*_{\langle \tau_0, \dots, \tau_{k-1} \rangle}$. Hence $\langle f^{p^*}_{\langle \tau_0, \dots, \tau_{k-1} \rangle}, A \cap A^{p^*} \rangle \in G$. Thus $A \supseteq \{ \tau_n \mid k \leq n < \omega \}$.

Assume $A \notin E(d)$. Then by a density argument we get an extension $q \leq p^*$ such that $q \in G$ and $(A^q \restriction f^{p^*}) \cap A = \emptyset$. Hence there is $k < \omega$ such that $q \leq^* p^*_{\langle \tau_0, \dots, \nu_{n-1} \rangle}$. Thus $\langle f^{p^*}_{\langle \tau_0, \dots, \tau_{n-1} \rangle}, A^{p^*} \setminus A \rangle \in G$. Hence for each $k \leq n < \omega$, $\tau_n \notin A$. □

Thus given a generic filter $G \subseteq \mathbb{P}$, the unboundedness of the set of elementary substructures for which F_G is N-generic is a necessary condition. To conclude the proof we will show that this condition is sufficient.

Assume $F : \lambda \setminus \kappa \to [\kappa]^\omega$ is a function. A condition $\langle f, A \rangle \in \mathbb{P}$ is said to be *F-generated* if there is a sequence $\langle \tau_n \mid n < \omega \rangle$ generating the measure $E(\operatorname{dom} f)$, $A \supseteq \{ \tau_n \mid n < \omega \}$, and for each $\alpha \in \operatorname{dom} f$,

$$F(\alpha) = \bigcup \{ f_{\langle \tau_0, \dots, \tau_n \rangle}(\alpha) \mid n < \omega \}.$$

Denote by $G_F \subseteq \mathbb{P}$ the set of F-generated conditions.

The following lemma holds in a universe extending V where F is a set.

Lemma 3.4. *Assume that the set* $\{N \in V \mid (N \prec H(\chi))_V,\ F \text{ is } N\text{-generic}\}$ *is unbounded in* $(H(\chi)^{\leq\kappa})_V$*. Then* G_F *is a generic filter.*

Proof. Assume $\langle f, A\rangle, \langle g, B\rangle \in G_F$. We will exhibit a condition $\langle h, C\rangle \in G_F$ such that $\langle h, C\rangle \leq \langle f, A\rangle, \langle g, B\rangle$.

By the unboundedness assumption there is an appropriate elementary substructure $N \prec H(\chi)$ in V such that $\langle f, A\rangle, \langle g, B\rangle \in N$ and F_G is N-generic. Hence there is an $\langle N, \mathbb{P}^*\rangle$-generic condition $h' \in \mathbb{P}^*$ and a sequence $\langle \tau'_n \mid n < \omega\rangle$ such that $F(\alpha) = \bigcup\{h'_{\tau'_0,\ldots,\tau'_n}(\alpha) \mid n < \omega\}$ for each $\alpha \in d$, and $\langle \tau'_n \mid n < \omega\rangle$ generates the measure $E(d)$, where $d = (\lambda \setminus \kappa) \cap N$.

Let $C \in E(d)$ be a good measure one set such that $C \restriction \operatorname{dom} f \subseteq A$ and $C \restriction \operatorname{dom} g \subseteq B$. Remove a measure zero set from C so that $\max h'(\alpha) < \nu(\kappa)$ will hold for each $\nu \in C$ and $\alpha \in \operatorname{dom}\nu$. Then there is $k < \omega$ such that $C \supseteq \{\tau'_n \mid k \leq n < \omega\}$. Set $h = h'_{\langle\tau'_0,\ldots,\tau'_{k-1}\rangle}$ and $\tau_n = \tau'_{k+n}$ for each $n < \omega$. Note $\langle h, C\rangle \in G_F$.

Let $\langle \rho_n \mid n < \omega\rangle$ be a sequence witnessing that the condition $\langle f, A\rangle$ is F-generated. We show that there are $k_0, k_2 < \omega$ such that $\tau_{k_2+n} \restriction \operatorname{dom} f = \rho_{k_0+n}$ for each $n < \omega$. Proceed as follows.

The sequences $\langle \rho_n(\kappa) \mid n < \omega\rangle$ and $\langle \tau_n(\kappa) \mid n < \omega\rangle$ are both tails of $F(\kappa)$. Hence there are $k_0, k_2 < \omega$ such that $\rho_{k_0+n}(\kappa) = \tau_{k_2+n}(\kappa)$ for each $n < \omega$. Thus for each $n < \omega$ we have

$$\rho_{k_0+n}(\kappa) = \tau_{k_2+n}(\kappa)$$

and

$$\rho_{k_0+n+1}(\kappa) = \tau_{k_2+n+1}(\kappa).$$

Fix $n < \omega$. Then $\tau_{k_2+n} \restriction \operatorname{dom} f \in A$. Since $\tau_{k_2+n}(\kappa) = \rho_{k_0+n}(\kappa)$, the last item of definition 2.1 yields $\operatorname{dom} \tau_{k_2+n} \restriction \operatorname{dom} f = \operatorname{dom} \rho_{k_0+n}$. Fix $\alpha \in \operatorname{dom} \rho_{k_0+n}$. Then

$$\rho_{k_0+n}(\kappa) \leq \rho_{k_0+n}(\alpha) < \rho_{k_0+n+1}(\kappa),$$

and

$$\tau_{k_2+n}(\kappa) \leq \tau_{k_2+n}(\alpha) < \tau_{k_2+n+1}(\kappa).$$

Thus both $\rho_{k_0+n}(\alpha)$ and $\tau_{k_2+n}(\alpha)$ are the unique ordinals in $F(\alpha)$ which are in the range $[\rho_{k_0+n}(\kappa), \rho_{k_0+n+1}(\kappa)) = [\tau_{k_2+n}(\kappa), \tau_{k_2+n+1}(\kappa))$. Hence $\rho_{k_0+n}(\alpha) = \tau_{k_2+n}(\alpha)$. Thus $\tau_{k_2+n} \restriction \operatorname{dom} f = \rho_{k_0+n}$.

Let $\langle \sigma_n \mid n < \omega\rangle$ be a sequence witnessing that the condition $\langle g, B\rangle$ is F-generated. Working as above we find $k_1 < \omega$ and enlarge k_2 if necessary so that $\tau_{k_2+n} \restriction \operatorname{dom} g = \sigma_{k_1+n}$ for each $n < \omega$.

Set $p = \langle f, A\rangle_{\langle\rho_0,\ldots,\rho_{k_0-1}\rangle}$, $q = \langle g, B\rangle_{\langle\sigma_0,\ldots,\sigma_{k_1-1}\rangle}$, and $r = \langle h, C\rangle_{\langle\tau_0,\ldots,\tau_{k_2-1}\rangle}$. By definition $p, q, r \in G_F$, $p \leq \langle f, A\rangle$, and $q \leq \langle g, B\rangle$. We will be done by showing that $r \leq p, q$.

Since $\langle \rho_{n_0+n} \mid n < \omega\rangle = \langle \tau_{n_2+n} \restriction \operatorname{dom} f \mid n < \omega\rangle$ we get $f^p = f^r \restriction \operatorname{dom} f$. Similarly $f^q = f^r \restriction \operatorname{dom} g$. It is clear that $C_{\langle\tau_0,\ldots,\tau_{k_2-1}\rangle} \restriction \operatorname{dom} f \subseteq A_{\langle\rho_0,\ldots,\rho_{k_0-1}\rangle}$ and $C_{\langle\tau_0,\ldots,\tau_{k_2-1}\rangle} \restriction \operatorname{dom} g \subseteq B_{\langle\sigma_0,\ldots,\sigma_{k_0-1}\rangle}$. Thus $r \leq p, q$.

We are left with proving the genericity of G_F. Let D be a dense open subset of $\mathbb{P}$. By the unboundedness assumption there is an appropriate elementary substructure $N \prec H(\chi)$ in V such that $D \in N$ and F is N-generic. Let the condition $f^* \in \mathbb{P}^*$

and the sequence $\langle \tau_n \mid n < \omega \rangle$ witness that F is N-generic. Set

$$D^* = \{f \in \mathbb{P}^* \mid \exists A \; \exists m < \omega \; \forall \langle \nu_0, \ldots, \nu_{m-1} \rangle \in [A]^m \; \langle f, A \rangle_{\langle \nu_0, \ldots, \nu_{m-1} \rangle} \in D\}.$$

Then $D^* \in N$ is a dense open subset of $\mathbb{P}^*$, and so $f^* \in D^*$. Thus there is a measure one set $A \in E(\operatorname{dom} f^*)$ and $m < \omega$ such that for each $\langle \nu_0, \ldots, \nu_{m-1} \rangle \in [A]^n$, $\langle f^*, A \rangle_{\langle \nu_0, \ldots, \nu_{n-1} \rangle} \in D$. In particular for each $m \leq n < \omega$, $\langle f^*, A \rangle_{\langle \tau_0, \ldots, \tau_{n-1} \rangle} \in D$. Finally, there is $k < \omega$ such that $A \supseteq \{\tau_n \mid k \leq n < \omega\}$. Hence $\langle f^*, A \rangle_{\langle \tau_0, \ldots, \tau_n \rangle} \in G_F$ for each $k \leq n < \omega$. We are done. □

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